THE CONSTITUTIVE RELATIONS AND THE MAGNETOELECTRIC EFFECT FOR MOVING MEDIA

Tomislav Ivezić
Ruđer Bošković Institute, P.O.B. 180, 10002 Zagreb, Croatia
E-mail: ivezic@irb.hr

In this paper the constitutive relations for moving media with homogeneous and isotropic electric and magnetic properties are presented as the connections between the generalized magnetization-polarization bivector $\mathcal{M}$ and the electromagnetic field F. Using the decompositions of F and $\mathcal{M}$, it is shown how the polarization vector $P(x)$ and the magnetization vector $M(x)$ depend on E, B and two different velocity vectors, u - the bulk velocity vector of the medium, and v - the velocity vector of the observers who measure E and B fields. These constitutive relations with four-dimensional geometric quantities, which correctly transform under the Lorentz transformations (LT), are compared with Minkowski's constitutive relations with the 3-vectors and several essential differences are pointed out. They are caused by the fact that, contrary to the general opinion, the usual transformations of the 3-vectors $\mathbf{E}$, $\mathbf{B}$, $\mathbf{P}$, $\mathbf{M}$, etc. are not the LT. The physical explanation is presented for the existence of the magnetoelectric effect in moving media that essentially differs from the traditional one.

Keywords: Constitutive relations; moving media; magnetoelectric effect.

1. Introduction

In this paper the constitutive relations for moving media with homogeneous and isotropic electric and magnetic properties are presented using the abstract four-dimensional (4D) geometric quantities and their representations in the standard basis. These results are compared with Minkowski's constitutive relations[1] with the 3-vectors and with Minkowski's constitutive relations that are obtained by means of exterior forms.[2] The paper is organized as follows.

First, in Sec. 2, we present a review of some previous results that are important for the theory presented here. In the recent paper,[3] a formulation of the field equations for moving media is developed by the generalization of an axiomatic geometric formulation of the electromagnetism in vacuum.[4] As mentioned in Ref. 3 almost the entire physics literature deals with the electromagnetic excitation tensor $\mathcal{H}$, i.e. with the electric D and magnetic H excitations (see Eq. (5)) and the constitutive relations refer to the connections between them and F, i.e. E and B, respectively. But, as shown in Ref. 3, it is physically better founded to formulate the field equations for moving media in terms of F and the generalized magnetization-polarization bivector $\mathcal{M}$ instead of, as usual, F

and $\mathcal{H}$. In Sec. 2, the decompositions of F (1) and $\mathcal{M}$ (3) are presented. F is decomposed into vectors E, B and v - the velocity vector of the observers who measure E and B fields. $\mathcal{M}$ is decomposed into the polarization vector P, the magnetization vector M and u - the bulk velocity vector of the medium. In Ref. 3, the field equations are written in terms of F and $\mathcal{M}$ and also in terms of vectors E, B, P, M and the velocity vectors u and v. These field equations are also quoted in Sec. 2. Furthermore, in Sec. 2, the usual transformations (UT) of the 3-vectors $\mathbf{E}$ and $\mathbf{B}$, (6), and of $\mathbf{P}$ and $\mathbf{M}$, (7), are written and their difference relative to the Lorentz transformations (LT) of vectors, as 4D geometric quantities, e.g., the electric field vector E, (8), is pointed out.

In Sec. 3, we formulate the constitutive relations as the relations between $\mathcal{M}$ and F, Eqs. (11) and (12). Then, using the decompositions of F (1) and $\mathcal{M}$ (3) we get from (11) and (12) how $P(x)$, (13), and $M(x)$, (14), depend on E, B and two different velocity vectors, u and v. The constitutive relations (13) and (14) are the basic relations that are obtained in this paper and they are not reported in previous approaches. The last term in (13) and (14) describes the magnetoelectric effect in a moving dielectric in a new way.

In Sec. 4, we have represented all 4D geometric quantities from (13) and (14) in the standard basis in order to compare them with some usual formulations. This procedure yields Eqs. (17) and (18).

In Sec. 5, Minkowski's constitutive relations with the 3-vectors (22) are quoted. The equations (22) are considered to be the fundamental constitutive equations for moving media in the whole physics community. In Sec. 5.1, the relations (22) are written in equivalent forms as constitutive equations which explicitly express the 3-vectors $\mathbf{P}$ and $\mathbf{M}$ as the functions of the 3-vectors $\mathbf{E}$ and $\mathbf{B}$, (23) and (24), or (25). These forms of Minkowski's constitutive relations are compared with our relations (17) and (18), i.e., with Eqs. (19) and (21), and several essential differences are pointed out. It is argued that these differences appear since Minkowski's constitutive relations are with the 3-vectors and they are derived using the UT and not the LT. In Sec. 5.2, it is shown that the same differences remain in the low velocity limit. In Sec. 5.3, it is presented the comparison with Minkowski's constitutive relations that are obtained by means of exterior forms.[2] It is shown that the constitutive relations with exterior forms from Ref. 2 are completely equivalent to Minkowski's constitutive relations with the 3-vectors and, accordingly, they also differ from the relations obtained in this paper.

In Sec. 6, Eq. (28) represents the interaction term in the Lagrangian for the interaction between the electromagnetic field F and the dipole moment bivector D, whereas Eq. (29) is its low velocity limit. The last two terms in (28), or (29), contain the direct interaction of E with the magnetic dipole moment vector m and B with the electric dipole moment vector d. These terms give the physical explanation for the existence of the magnetoelectric effect in moving media. That explanation markedly differs from the traditional one.

In Sec. 7, some remarks are given, which refer to the general constitutive relations.

In Sec. 8, the conclusions are presented.

2. A Short Review of Some Previous Results

We shall deal with 4D geometric quantities, i.e. in the geometric algebra formalism. For the exposition of the geometric algebra see Ref. 5. The generators of the spacetime algebra are four basis vectors $\{\gamma_\mu\}, \mu = 0...3$, satisfying $\gamma_\mu \cdot \gamma_\nu = \eta_{\mu\nu} = diag(+---)$. This basis, the standard basis, is a right-handed orthonormal frame of vectors in the Minkowski spacetime M^4 with γ_0 in the forward light cone, $\gamma_0^2 = 1$ and $\gamma_k^2 = -1$ $(k = 1, 2, 3)$. The standard basis $\{\gamma_\mu\}$ corresponds to Einstein's system of coordinates in which the Einstein synchronization of distant clocks[6] and Cartesian space coordinates x^i are used in the chosen inertial frame of reference.

First, we briefly expose the formulation of the field equations from Ref. 3. As shown in Ref. 3, the field equations (5), $\partial(\varepsilon_0 F + \mathcal{M}) = j^{(C)}/c$; $\partial \cdot (\varepsilon_0 F + \mathcal{M}) = j^{(C)}/c$, $\partial \wedge F = 0$, can be taken as the primary equations for the electromagnetism in moving media. The bivector $F = F(x)$ represents the electromagnetic field and, as shown in Ref. 4, it can be taken as the primary quantity for the whole electromagnetism. $j^{(C)}$ is the conduction current density of the *free* charges. $\mathcal{M}$ is the generalized magnetization-polarization bivector $\mathcal{M} = \mathcal{M}(x)$, which is connected with the magnetization-polarization current density of the *bound* charges $j^{(\mathcal{M})} = -c\partial\mathcal{M} = -c\partial \cdot \mathcal{M}$. (According to Eq. (3),[3] the total current density vector j can be decomposed as $j = j^{(C)} + j^{(\mathcal{M})}$.) The field equation with sources is written in the 'source representation' in Eq. (7)[3] $\partial \cdot \varepsilon_0 F = j^{(C)}/c - \partial \cdot \mathcal{M}$; the sources of F are the true currents $j^{(C)}$ and the magnetization-polarization current density $\partial \cdot \mathcal{M}$. In most materials $\mathcal{M}$ is a function of the field F and this dependence is determined by the constitutive relations. In all previous formulations of the electromagnetism in media (at rest, or moving), the electromagnetic excitation tensor is introduced as $\mathcal{H} = \varepsilon_0 F + \mathcal{M}$. Then, the constitutive relations refer to the connections between $\mathcal{H}$ and F. However, as discussed in Ref. 3, physically different kind of entities are mixed in that definition for $\mathcal{H}$; an electromagnetic field F and a matter field $\mathcal{M}$. Furthermore, in general, two different velocity vectors, v - the velocity of the observers and u - the velocity of the moving medium, enter into the decompositions of F and $\mathcal{M}$, respectively. For F, that decomposition is given by Eq. (10)[3]

$$F = E \wedge v/c + (IcB) \cdot v/c, \tag{1}$$

where the electric and magnetic fields are represented by vectors $E(x)$ and $B(x)$ and I is the unit pseudoscalar. There is no rest frame for the field F, that is, for E and B, and therefore the vector v in the decomposition (1) is interpreted as the velocity vector of the observers who measure E and B fields. Then $E(x)$ and $B(x)$ are defined with respect to v, i.e., with respect to the observer, as

$$E = F \cdot v/c, \quad B = -(1/c)I(F \wedge v/c). \tag{2}$$

It also holds that $E \cdot v = B \cdot v = 0$; both E and B are space-like vectors and they depend not only on F but on v as well. Similarly, in Eq. (12),[3] the bivector

$\mathcal{M}(x)$ is decomposed into two vectors, the polarization vector $P(x)$ and the magnetization vector $M(x)$ and the unit time-like vector u/c

$$\mathcal{M} = P \wedge u/c + (MI) \cdot u/c^2. \tag{3}$$

There is the rest frame for a medium, i.e., for $\mathcal{M}$, or P and M, and therefore the vector u in the decomposition (3) is identified with bulk velocity vector of the medium in spacetime. Then, $P(x)$ and $M(x)$ are defined with respect to u as

$$P = \mathcal{M} \cdot u/c, \quad M = cI(\mathcal{M} \wedge u/c) \tag{4}$$

and it holds that $P \cdot u = M \cdot u = 0$. As in the case with F, it is visible from (4) that P and M depend not only on $\mathcal{M}$ but on u as well.

Usually, only the velocity vector u of the moving medium is taken into account, or the case $u = v$ is considered,[7] i.e., it is supposed that the observer frame is comoving with medium, or both decompositions (1) and (3) are made with the same velocity vector, either u or v.[8] Such assumptions enable the introduction of the electromagnetic excitation bivector $\mathcal{H}$, and, by using (1) and (3), one finds the decomposition of $\mathcal{H}$ into the electric and magnetic excitations (other names of which are 'electric displacement' and 'magnetic field intensity')

$$\mathcal{H} = D \wedge u/c + (IH) \cdot u/c^2 \tag{5}$$

(Eq. (14)[3]), where, as usual, the electric displacement vector $D = \varepsilon_0 E + P$ and the magnetic field intensity vector $H = (1/\mu_0)B - M$ are introduced.

In Ref. 3, inserting the decompositions of F and $\mathcal{M}$ (Eqs. (1) and (3) here) into the field equation, the general form of the field equation for a magnetized and polarized moving medium with $E(x)$, $B(x)$, $P(x)$ and $M(x)$ (the Ampèrian form) is obtained in the form of Eq. (15), i.e., (16) (the vector part, with sources), or (18) in the 'source representation' $\partial \cdot \{\varepsilon_0[E \wedge v/c + (IB) \cdot v]\} = j^{(C)}/c - \partial \cdot [P \wedge u/c + (1/c^2)(MI) \cdot u]$, and (17) (the trivector part, without sources) $\partial \wedge [E \wedge v/c + (IB) \cdot v] = 0$. In contrast to all previous results, in the vector part, i.e., the part with sources, of the field equation there are two different velocities u and v. From these field equations it is concluded[3] that in the field equation with sources, (16) or (18) in Ref. 3, *the usual Ampèr-Maxwell law and Gauss's law are inseparably connected in one law.* Similarly, *Faraday's law and the law that expresses the absence of magnetic charge are also inseparably connected in one law,* the field equation without sources, Eq. (17).[3] As shown in Secs. 6 and 7 in Ref. 3, this inseparability is an essential difference relative to the usual Maxwell equations with the 3-vectors.

Next, we mention an important result regarding the usual formulation of electromagnetism, as in Ref. 9, which is presented in Ref. 10 and discussed in Refs. 11, 12 and 3. It is argued[10] that an individual vector has no dimension; the dimension is associated with *the dimension of its domain.* Hence, the time-dependent $\mathbf{E}(\mathbf{r},t)$, $\mathbf{B}(\mathbf{r},t)$, $\mathbf{D}(\mathbf{r},t)$ etc. cannot be the 3-vectors, since they are defined on the spacetime. Therefore, we use the term 'vector' for a geometric quantity, which is defined on the spacetime and which always has in some basis

of that spacetime, e.g., the standard basis $\{\gamma_\mu\}$, four components (some of them can be zero). Note that vectors are usually called the 4-vectors. However, an incorrect expression, the 3-vector, will still remain for the usual $\mathbf{E}(\mathbf{r},t)$, $\mathbf{B}(\mathbf{r},t)$, $\mathbf{D}(\mathbf{r},t)$ etc.. Moreover, recently,[13−17,11] it is proved that, contrary to the general belief, the UT of the 3-vectors of the electric and magnetic fields, $\mathbf{E}(\mathbf{r},t)$ and $\mathbf{B}(\mathbf{r},t)$ respectively, see, e.g., Eqs. (11.148) and (11.149) in Ref. 9, differ from the LT (boosts) of the corresponding 4D quantities that represent the electric and magnetic fields. As explained,[13−17,11] the fundamental difference between the UT and the LT of the electric and magnetic fields is that in the UT, e.g., the components of the transformed $\mathbf{E}'$ are expressed by the mixture of components of $\mathbf{E}$ and $\mathbf{B}$, and similarly for $\mathbf{B}'$, Eq. (11.148).[9] The UT of the 3-vectors $\mathbf{E}$ and $\mathbf{B}$ are given, e.g., by Eqs. (11.149)[9] and they are

$$\begin{aligned}
\mathbf{E}' &= \gamma(\mathbf{E} + \beta\times c\mathbf{B}) - (\gamma^2/(1+\gamma))\beta(\beta\cdot\mathbf{E}),\\
\mathbf{B}' &= \gamma(\mathbf{B} - (1/c)\beta\times\mathbf{E}) - (\gamma^2/(1+\gamma))\beta(\beta\cdot\mathbf{B}),
\end{aligned} \tag{6}$$

where $\mathbf{E}'$, $\mathbf{E}$, β and $\mathbf{B}'$, $\mathbf{B}$ are all 3-vectors. All what is stated for the 3-vectors $\mathbf{E}$ and $\mathbf{B}$ and their UT holds in the same measure for the couple of the 3-vectors $\mathbf{P}$ and $\mathbf{M}$ and their UT

$$\begin{aligned}
\mathbf{P} &= \gamma(\mathbf{P}' + \beta\times\mathbf{M}'/c) - (\gamma^2/(1+\gamma))\beta(\beta\cdot\mathbf{P}'),\\
\mathbf{M} &= \gamma(\mathbf{M}' - \beta\times c\mathbf{P}') - (\gamma^2/(1+\gamma))\beta(\beta\cdot\mathbf{M}'),
\end{aligned} \tag{7}$$

see the equations, e.g., Eq. (4.2),[18] or Eqs. (18-68) - (18-71),[19] or (6.78a) and (6.81a),[20] etc.

However, *the correct LT always transform the 4D algebraic object (vector, bivector) representing the electric field only to the electric field, and similarly for the magnetic field.*

In order to explain this fundamental difference between the LT and the UT let us introduce the frame of 'fiducial' observers as the frame in which the observers who measure fields E and B are at rest. That frame with the standard basis $\{\gamma_\mu\}$ in it is called the γ_0-frame. In the γ_0-frame $v = c\gamma_0$ and therefore E from (2) becomes $E = F\cdot\gamma_0$ and it transforms under the active LT (Eqs. (10) and (11) in Ref. 11) in such a manner that both F and the velocity of the observer $v = c\gamma_0$ are transformed by the LT, see Eq. (12)[11] ($E = F\cdot\gamma_0 \longrightarrow E' = R(F\cdot\gamma_0)\widetilde{R} = (RF\widetilde{R})\cdot(R\gamma_0\widetilde{R})$). As explained in Ref. 11, *Minkowski, in Sec.* 11.6,[1] *showed that both factors of the vector E, as the product of one bivector and one vector, has to be transformed by the LT.* However, it is worth mentioning that Minkowski in all other parts of Ref. 1 dealt with the usual 3-vectors $\mathbf{E}$, $\mathbf{B}$, $\mathbf{D}$, etc.. These correct LT give that

$$E' = E + \gamma(E\cdot\beta)\{\gamma_0 - (\gamma/(1+\gamma))\beta\}, \tag{8}$$

Eq. (13).[11] *In the same way vector B transforms and vectors P, M as well, but for P and M the LT, like* (8), *are the transformations from the rest frame of*

the medium ($u = c\gamma_0$). For boosts in the direction γ_1 one has to take in that equation that $\beta = \beta\gamma_1$ (on the l.h.s. is vector β and on the r.h.s. β is a scalar). Hence, in the standard basis and when $\beta = \beta\gamma_1$ that equation becomes

$$E'^{\nu}\gamma_\nu = -\beta\gamma E^1\gamma_0 + \gamma E^1\gamma_1 + E^2\gamma_2 + E^3\gamma_3, \tag{9}$$

what is Eq. (14).[11] The most important result is that *the electric field vector E transforms by the LT again to the electric field vector E'; there is no mixing with the magnetic field B.* The same happens with vectors P and M.

On the other hand, if in the transformation of $E = F \cdot \gamma_0$ only F is transformed by the LT R, but not the velocity of the observer $v = c\gamma_0$ ($E = F \cdot \gamma_0 \longrightarrow E'_F = (RF\widetilde{R}) \cdot \gamma_0$, Eq. (15)[11]), then, in the standard basis and when $\beta = \beta\gamma_1$, one finds

$$E'^{\nu}_F\gamma_\nu = E^1\gamma_1 + \gamma(E^2 - c\beta B^3)\gamma_2 + \gamma(E^3 + c\beta B^2)\gamma_3. \tag{10}$$

what is Eq. (17).[11] It is visible from the comparison of Eq. (10) with Eq. (11.148)[9] that the transformations of components (taken in the standard basis) of E'_F are exactly the same as the transformations of $E_{x,y,z}$ from Eq. (11.148),[9] i.e., the components from (6).

3. Constitutive Relations for Moving Media in Geometric Terms

Let us consider the case of a simple medium with homogenous and isotropic electric and magnetic properties. In accordance with Sec. 2 we formulate the constitutive relations for moving media using the generalized magnetization-polarization bivector $\mathcal{M}$ and the electromagnetic field bivector F

$$\mathcal{M} \cdot u = \varepsilon_0\chi_E F \cdot u, \tag{11}$$

$$(I\mathcal{M}) \cdot u = (\chi_B/\mu_0 c^2) u \cdot (IF) \tag{12}$$

and the electric and magnetic susceptibility (χ_E, χ_B).

Using the decompositions of F (1) and $\mathcal{M}$ (3) we get from (11) how the polarization vector $P(x)$ depends on E, B and u, v,

$$P = (\varepsilon_0\chi_E/c)\{(1/c)[(u \cdot v)E - (u \cdot E)v] + (u \wedge v \wedge B)I\} \tag{13}$$

and from (12) how the magnetization vector $M(x)$ depends on E, B and u, v,

$$M = \varepsilon_0\chi_B\{[(u \cdot v)B - (u \cdot B)v] - (1/c)(u \wedge v \wedge E)I\}. \tag{14}$$

Observe that P in (13) and M in (14) contain both velocities u, the bulk velocity vector of the medium, and v, the velocity vector of the observers. The relations (13) and (14) are the basic relations that are obtained in this paper. In our approach, the relations (13) and (14) replace the constitutive relations with the 3-vectors (23) and (24), which are equivalent to Minkowski's equations (22). The equations (23), (24) and (22) are given below in Sec. 5.

In the special case when $v = u$, an observer frame comoving with medium, (13) and (14) yield

$$P = \varepsilon_0 \chi_E E, \qquad M = (\chi_B/\mu_0)B, \tag{15}$$

which are the familiar linear constitutive laws for the case of an electrically neutral, isotropic, non-dispersive, polarizable medium at rest. Introducing the relative permittivity ε_r, $\varepsilon_r = 1 + \chi_E$, and the relative permeability μ_r, $\mu_r = 1/(1 - \chi_B)$, the constitutive laws (15) can be written as $P = \varepsilon_0(\varepsilon_r - 1)E$, $M = (1/\mu_0)(1 - 1/\mu_r)B$. In this case ($v = u$) the excitations D, $D = \varepsilon_0 E + P$ and H, $H = (1/\mu_0)B - M$ can be introduced, as in Sec. 2, and the rest frame constitutive relations take the usual forms

$$D = \varepsilon E, \qquad H = (1/\mu)B, \tag{16}$$

where $\varepsilon = \varepsilon_0 \varepsilon_r$ and $\mu = \mu_0 \mu_r$.

The last term in (13) and (14) describes the magnetoelectric effects in a moving dielectric. According to the last term in (13) a moving dielectric becomes electrically polarized when placed in a magnetic field, the Wilsons' experiment.[21] Similarly, according to the last term in (14) a moving dielectric becomes magnetized when it is placed in an electric field, Röntgen's experiment.[22]

4. Constitutive Relations for Moving Media in the Standard Basis

The equations (11) - (16) are all coordinate-free relations. In order to compare them with some usual formulations we have to represent all 4D geometric quantities from them in the standard basis $\{\gamma_\mu\}$. Then, in the $\{\gamma_\mu\}$ basis, Eq. (13) becomes

$$P^\mu \gamma_\mu = (\varepsilon_0 \chi_E/c)[(1/c)(E^\mu v^\nu - E^\nu v^\mu) + \varepsilon^{\mu\nu\alpha\beta} v_\alpha B_\beta] u_\nu \gamma_\mu, \tag{17}$$

whereas (14) takes the form

$$M^\mu \gamma_\mu = \varepsilon_0 \chi_B [(B^\mu v^\nu - B^\nu v^\mu) + (1/c)\varepsilon^{\mu\nu\alpha\beta} E_\alpha v_\beta] u_\nu \gamma_\mu. \tag{18}$$

We shall examine Eqs. (17) and (18) taking that the laboratory frame, the S frame, is the γ_0-frame ($v = c\gamma_0$, $v^\mu = (c, 0, 0, 0)$, $E^0 = B^0 = 0$) in which the material medium, the S' frame, is moving with velocity u, $u^\nu = (\gamma_u c, \gamma_u U^1, \gamma_u U^2, \gamma_u U^3)$, U^k are the components of the 3-velocity $\mathbf{U}$ and $\beta_u = |\mathbf{U}|/c$. Then, in the laboratory frame, Eq. (17) becomes

$$P = \varepsilon_0 \chi_E \gamma_u [E^i (U^i/c)\gamma_0 + (E^i + \varepsilon^{0ijk} U_j B_k)\gamma_i]. \tag{19}$$

It can be seen from (19) that, e.g., if $E^\mu = (0, 0, 0, 0)$, $B^\mu = (0, 0, 0, -B^3)$, $u^\mu = (\gamma_u c, \gamma_u U^1, 0, 0)$, the components of the polarization are

$$P^\mu = (0, 0, P^2 = \varepsilon_0 \chi_E \gamma_u U^1 B^3, 0); \tag{20}$$

these components correspond to the 'translational' version of Wilsons' experiment. As stated on page 13,[23] the magnetoelectric effect 'in a moving dielectric is of course anisotropic in the sense that the polarization depends upon an applied field which is perpendicular to the direction of motion and this polarization is then perpendicular to both the applied field and the direction of motion.' Comparing P^2 from (20) with P_y from Eq. (1.14)[23] (see also the term $\chi_{(em)}^{\alpha\beta}$ in Fig. 3.3 and Eq. (2.9)[23]), we see that for the above conditions (the electric field is absent) these two expressions differ only in the γ_u factor; P^2 contains γ_u whereas P_y contains γ_u^2.

However, it is worth mentioning that the complete P_y (both, the 3-vectors of the electric and magnetic fields exist) from Eq. (1.14),[23] or the components of the 3-vector $\mathbf{P}$ from Eq. (2.9) and Fig. 3.3,[23] can be compared only with the spatial components of (19), but P in (19) contains the temporal component as well. Again, the *spatial components* of (19) and the complete P_y differ only in the γ_u factor.

Similarly, from (18), we find

$$M = (\chi_B/\mu_0)\gamma_u[B^i(U^i/c)\gamma_0 + (B^i - \varepsilon^{0ijk}U_j E_k/c^2)\gamma_i]. \tag{21}$$

In the case of low velocities of the medium, $\beta_u \ll 1$, the equations (19) and (21) are almost unchanged to first order of β_u; only it is taken that $\gamma_u \simeq 1$. Observe that in (19) and (21) the terms with γ_0 are comparable to the terms containing $U_j B_k \gamma_i$ and $(1/c^2)U_j E_k \gamma_i$, respectively. Thus, to first order of β_u, the terms P^0 and M^0 *cannot be neglected* relative to the complete P^i and M^i, respectively.

The constitutive relations (19) and (21) significantly differ from all previous constitutive equations for such simple moving media. They will be compared with the usual formulations with the 3-vectors.

5. Comparison with Minkowski's Constitutive Relations and with Their Derivations

Let us examine some of the usual derivations of the constitutive relations for moving media. As already mentioned, almost the entire physics literature deals with the electromagnetic excitation tensor $\mathcal{H}$, i.e. with the electric D and magnetic H excitations and the constitutive relations refer to the connections between them and F, i.e. E and B, respectively.

Minkowski[1] was the first who derived the constitutive relations for the 3-vectors

$$\begin{aligned} \mathbf{D} + (1/c^2)\mathbf{U}\times\mathbf{H} &= \varepsilon(\mathbf{E} + \mathbf{U}\times\mathbf{B}) \\ \mathbf{B} - (1/c^2)\mathbf{U}\times\mathbf{E} &= \mu(\mathbf{H} - \mathbf{U}\times\mathbf{D}), \end{aligned} \tag{22}$$

Eqs. (C) and (D) in Sec. 8.[1] From that time, the equations (22) are considered to be the fundamental constitutive equations for moving media in the whole

physics community. They are subsequently derived in different ways and used in numerous papers and textbooks on the electromagnetism.

5.1 Comparison with Minkowski's constitutive relations that are obtained by means of the 3-vectors[24] and in the covariant approach[25]

One way of the derivation of Eqs. (22) is presented in Ref. 24. There, these equations, i.e. Eqs. (7),[24] are derived using Minkowski's crucial hypothesis that the relations with the 3-vectors $\mathbf{D} = \varepsilon\mathbf{E}$, $\mathbf{H} = (1/\mu)\mathbf{B}$, which correspond to (16), retain their form in the moving frame S' with the same ε and μ, i. e., $\mathbf{D}' = \varepsilon\mathbf{E}'$, $\mathbf{H}' = (1/\mu)\mathbf{B}'$. Then, in Ref. 24, *the UT* (6) *and the similar UT for* $\mathbf{D}'$ *and* $\mathbf{H}'$ (Eqs. (3)[24]) *are used to obtain Minkowski's constitutive equations* (22). Furthermore, in Ref. 24, the constitutive relations, which explicitly express $\mathbf{D}$ and $\mathbf{H}$ in terms of $\mathbf{E}$ and $\mathbf{B}$, are derived (Eqs. (10)[24]) from (22). Introducing $\mathbf{D} = \varepsilon_0\mathbf{E} + \mathbf{P}$ and $\mathbf{H} = (1/\mu_0)\mathbf{B} - \mathbf{M}$ into Eqs. (10)[24] we find the constitutive equations which explicitly express $\mathbf{P}$ as a function of $\mathbf{E}$ and $\mathbf{B}$

$$\begin{aligned}\mathbf{P} &= \gamma^2\varepsilon_0\{\chi_E[\mathbf{E} - c^{-2}\mathbf{U}(\mathbf{U}\cdot\mathbf{E}) + \mathbf{U}\times\mathbf{B}] \\ &\quad +\chi_B[\mathbf{U}\times(\mathbf{B} - c^{-2}\mathbf{U}\times\mathbf{E})]\}\end{aligned} \tag{23}$$

and also $\mathbf{M}$ in terms of $\mathbf{E}$ and $\mathbf{B}$

$$\begin{aligned}\mathbf{M} &= (\gamma^2/\mu_0)\{\chi_B[\mathbf{B} - c^{-2}\mathbf{U}(\mathbf{U}\cdot\mathbf{B}) - c^{-2}\mathbf{U}\times\mathbf{E}] \\ &\quad -c^{-2}\chi_E[\mathbf{U}\times(\mathbf{E} + \mathbf{U}\times\mathbf{B})]\}.\end{aligned} \tag{24}$$

The equations (23) and (24) are equivalent to Minkowski's equations (22).

Introducing the relative permittivity ε_r and the relative permeability μ_r instead of the susceptibilities χ_E and χ_B the equations (23) and (24) become

$$\begin{aligned}\mathbf{P} &= \varepsilon_0(\varepsilon_r - 1)\mathbf{E} + \gamma^2\varepsilon_0(\varepsilon_r - 1/\mu_r)\mathbf{U}\times(\mathbf{B} - c^{-2}\mathbf{U}\times\mathbf{E}), \\ \mathbf{M} &= \mu_0^{-1}(1 - 1/\mu_r)\mathbf{B} - \gamma^2\varepsilon_0(\varepsilon_r - 1/\mu_r)\mathbf{U}\times(\mathbf{E} + \mathbf{U}\times\mathbf{B}).\end{aligned} \tag{25}$$

and they are, in the same way as Eqs. (23) and (24), equivalent to Minkowski's equations (22).

Similarly, in Ref. 25, the relations (22) (Eqs. (76.9)[25]) are obtained by the covariant generalization (only components, implicitly taken in the standard basis) of the relations with the 3-vectors $\mathbf{D} = \varepsilon\mathbf{E}$, $\mathbf{H} = (1/\mu)\mathbf{B}$, which correspond to our Eq. (16). The covariant generalizations from Ref. 25 are $\mathcal{H}^{\lambda\mu}u_\mu = \varepsilon F^{\lambda\mu}u_\mu$ ((76.7)[25]) and ${}^*F^{\lambda\mu}u_\mu = \mu{}^*\mathcal{H}^{\lambda\mu}u_\mu$ ((76.8)[25]), where ${}^*F^{\lambda\mu}$ and ${}^*\mathcal{H}^{\lambda\mu}$ are the dual tensors, i.e., only the components (${}^*F^{\alpha\beta} = (1/2)\varepsilon^{\alpha\beta\gamma\delta}F_{\gamma\delta}$). Then, Minkowski's constitutive equations (22) with the 3-vectors are derived by *Minkowski's identifications,*[1] *of components of* $F^{\alpha\beta}$ *with components of the 3-vectors* $\mathbf{E}$ *and* $\mathbf{B}$ ($E^i = F^{i0}$, $B^i = (1/2c)\varepsilon^{ijk0}F_{jk}$, see, e.g., Eq. (11.137)[9]) *and by the similar identifications for* $\mathcal{H}^{\alpha\beta}$ *and the 3-vectors* $\mathbf{D}$ *and* $\mathbf{H}$ ($D^i = \mathcal{H}^{i0}$, $H^i = (c/2)\varepsilon^{ijk0}\mathcal{H}_{jk}$).

However, as discussed, e.g. in Sec. 2,[11] the components are not the whole physical quantity. The mentioned identifications of components are synchronization dependent and the UT (6) (and the similar ones for $\mathbf{D}'$ and $\mathbf{H}'$) significantly differ from the LT (8).

On the other hand, our equations (11) and (12) deal with coordinate-free, 4D geometric quantities. Instead of Minkowski's identifications of components, and the same ones in Ref. 25, the mathematically correct decompositions of F (1) and $\mathcal{M}$ (3) are used for the derivation of the constitutive equations (13) and (14) (and, in the standard basis, (17) and (18)) from (11) and (12), respectively.

As mentioned above, the equations (23) and (24), or (25), which are equivalent to (22), can be compared with Eqs. (19) and (21). There are important differences between them.

1) The equations (19) and (21) are with correctly defined 4D quantities that properly transform under the LT, whereas it is not the case with Eqs. (23) and (24), or (25), with the 3-vectors that transform according to the UT.

2) Furthermore, (19) and (21) contain the term with γ_0, which cannot exist in the approach with the 3-vectors, i.e. in (23) and (24), or (25).

3) Also, in (19), the polarization vector P contains only the electric susceptibility χ_E and not χ_B, whereas, in (23), the polarization 3-vector $\mathbf{P}$ contains both susceptibilities χ_E and χ_B, (or, in (25), both, ε_r and μ_r). Similarly, in (21), there is only χ_B and not χ_E, whereas, in (24), there are both susceptibilities χ_B and χ_E, (or, in (25), both, μ_r and ε_r). It can be seen from the derivation of (23) and (24), or (25), that both susceptibilities appear in the expressions for $\mathbf{P}$ (23) and $\mathbf{M}$ (24) because the UT (6) and the similar ones for $\mathbf{D}'$ and $\mathbf{H}'$ (Eqs. (3)[24]), or, equivalently, the UT (7), are used and in them, e.g., in (6), $\mathbf{E}'$ *is expressed by the mixture of* $\mathbf{E}$ *and* $\mathbf{B}$, and similarly, in (7), $\mathbf{P}'$ *is expressed by the mixture of* $\mathbf{P}$ *and* $\mathbf{M}$.

5.2. Comparison of the low velocity limits

These differences remain in the low velocity limit, $\beta_u \ll 1$, as well. That limit is already examined for vectors P and M from (19) and (21), respectively. Only $\gamma_u \simeq 1$ is taken in (19) and (21).

In Ref. 24, the low velocity limit, i.e., the quasi-static approximation, is obtained in the same way by putting $\gamma_u \simeq 1$ in Eqs. (10) for $\mathbf{D}$ and $\mathbf{H}$ expressed in terms of $\mathbf{E}$ and $\mathbf{B}$, which led to Eqs. (11).[24] They, Eqs. (11),[24] are commonly used in literature. In the formulation with the 3-vectors $\mathbf{P}$ and $\mathbf{M}$ the equations (25) are derived from Eqs. (10).[24] Therefore, in that approximation, the usual 3-vectors $\mathbf{P}$ and $\mathbf{M}$ are given by (25) but with $\gamma_u \simeq 1$. This means that again both ε_r and μ_r, i.e., both susceptibilities χ_E and χ_B, appear in the quasi-static approximation in the expressions for $\mathbf{P}$ and $\mathbf{M}$. In Ref. 24, it is argued that such a procedure with $\gamma_u \simeq 1$ does not give a correct quasi-static approximation, because the obtained equations, (11),[24] 'do not obey the group additivity property.' Hence, according to Ref. 24, the equations (11)[24] have to be replaced by two well-defined Galilean limits of Minkowski's constitutive

equations ((22) here), the magnetic and electric limits, i.e. with two sets of low-velocity formulae. These two sets are Eqs. (12) and (13) in Ref.24 and it is stated there: 'Both sets (12) and (13) do form a group.'

However, the UT (6) and (7) and the similar UT for $\mathbf{D}'$ and $\mathbf{H}'$ (Eqs. (3)[24]) are not the LT and therefore constitutive equations with the 3-vectors, (22), or Eqs. (10),[24], and also (23) and (24), or (25), here, are not the *relativistic* constitutive equations and also Eqs. (12) and (13)[24] are not a quasi-static approximation of the relativistically correct constitutive relations. Thus, as stated above, all differences from Sec. 5.1 remain in the low velocity limit as well.

In the special case of moving ($\beta_u \ll 1$) nonmagnetic ($\mu_r = 1$, i.e. $\chi_B = 0$) media the 3D polarization $\mathbf{P}$ from (25), to first order of β_u, becomes

$$\mathbf{P} = \varepsilon_0 \chi_E (\mathbf{E} + \mathbf{U} \times \mathbf{B}). \tag{26}$$

The equation (26) is Eq. (9-19) (2),[19] or the last equation in Problem 6.8.[20]

But, even for $\beta_u \ll 1$, P from (19) will have the same form for both cases $\chi_B \neq 0$ and $\chi_B = 0$, because it does not depend on χ_B ($P \simeq \varepsilon_0 \chi_E [E^i (U^i/c)\gamma_0 + (E^i + \varepsilon^{0ijk} U_j B_k)\gamma_i]$). As already stated, to first order of β_u, P^0 cannot be neglected relative to P^i, which means that we have not an equation that would correspond to (26).

5.3. Comparison with constitutive equations that are obtained by means of exterior forms[2]

It is worth noting that Minkowski's constitutive equations (22) are also obtained in Ref. 2 using exterior calculus, i.e., with the use of abstract 4D geometric quantities, Eqs. (E.4.28) and (E.4.29).[2] At that place (p. 353) Hehl and Obukhov stated: 'Originally, the constitutive relations (E.4.28), (E.4.29) were derived by Minkowski with the help of Lorentz transformations for the case of a flat space time and a uniformly moving medium. We stress, however, that the Lorentz group never entered the scene in our derivation above.' It can be concluded from these statements that the authors[2] believe, as all others, that the UT (6) are the relativistically correct LT. However, as discussed in Sec. 2, the LT are given by Eq. (8) and not by (6). Furthermore, in Ref. 2, the expressions for the polarization P and the magnetization M, the equations (E.4.30) and (E.4.31), respectively, are derived from the constitutive relations (E.4.28), (E.4.29). The expressions for P and M, (E.4.30) and (E.4.31), respectively, from Ref. 2, are very similar to the equations with the 3-vectors, (23) and (24), respectively, which are obtained from (22). Indeed, all quantities in the constitutive relations (E.4.28), (E.4.29) and in the expressions for P and M, (E.4.30) and (E.4.31), respectively, from Ref. 2, are in the 3D subspace, i.e., like the usual 3-vectors. (Observe that v that enters into these relations is the 3-velocity 1-form, (E.4.27), or the 3-velocity vector. Also, the Hodge star operators in these expressions are defined by the 3-space metrics of the laboratory and material foliations.) This

means that these results[2] also significantly differ from the constitutive relations (13) and (14) in which all quantities are the 4D geometric quantities and there is no the space-time split. Let us describe in more detail the calculation from Ref. 2, which led to the mentioned results.[2]

In Ref. 2, Minkowski's constitutive relations (E.4.28), (E.4.29) and the expressions for P and M, (E.4.30) and (E.4.31), respectively, are derived using (1+3) - splitting of spacetime both in the laboratory frame and in the frame of moving macroscopic matter, the material frame. The starting relations in that derivation are the decompositions ((1+3) - splitting) of the electromagnetic excitation tensor $\mathcal{H}$ and of F in the laboratory frame (unprimed quantities), ($\mathcal{H} = -H \wedge dt + D$, $F = E \wedge dt + B$) Eq. (E.4.14), and in the material frame (primed quantities), ($\mathcal{H} = -H' \wedge dt' + D'$, $F = E' \wedge dt' + B'$) Eq. (E.4.15). (In both equations our notations is used.) There is an assumption in connection with the decompositions (E.4.14) and (E.4.15) in Ref. 2. It is: 'Clearly, we preserve the same symbols $\mathcal{H}$ (our notation) and F on the left-hand sides of (E.4.14) and (E.4.15) because these are just the same physical objects. In contrast, the right-hand sides are of course different, hence we use primes.'

From the mathematical point of view there are no reasons for such argumentations. If H, dt, D, E, B are transformed then $\mathcal{H}$ and F have to be transformed as well, as can be concluded going to a uniformly moving medium, i.e., when the LT are applicable. Let us explain these assertions in more detail. *There is no mathematical procedure by which in one equation some parts of it are transformed and the other parts are not. It is not possible that there is one law for the transformations of some 4D geometric quantities and another law for the transformations of the other quantities.* As mentioned, e.g., in Ref. 11, "*any multivector M transforms by the active LT in the same way*, Eq. (11),[11] $M \to M' = RM\widetilde{R}$, where the Lorentz boost R is given by Eq. (10).[11] Hence, if R is applied to (1), $R(F = E \wedge v/c + (IcB) \cdot v/c)\widetilde{R}$, then F *is also transformed*, together with E, B and v. Thus, the assertion that F is unchanged under the active LT is without any mathematical justification.

In addition, let us compare the relations (E.4.14) and (E.4.15) with the consideration from Ref. 26. In Sec. 4 'Premetric electrodynamics in exterior calculus' in Ref. 26, it is argued that the decomposition of $\mathcal{H}$, Eq. (31),[26] if put in matrix form, yields the identification $(6)_1$ in Ref. 26. Furthermore, the field strength F is decomposed in the same way as $\mathcal{H}$, i.e. 'into two pieces: one along the 1-dimensional time t and another one embedded in 3-dimensional space $(a, b = 1, 2, 3.)$.' Thus, in the notation,[26] $F = E \wedge dt + B = E_a dx^a \wedge dt + (1/2)B_{ab} dx^a \wedge dx^b$, Eq. (34).[26] If put in matrix form, it yields the usual identification $(6)_2$ in Ref. 26, i.e., in our notation, $E^i = F^{i0}$, $B^i = (1/2c)\varepsilon^{ijk0} F_{jk}$. The decompositions (31) and (34)[26] are the same as the decompositions (E.4.14).[2] As already stated, in the material frame, the same decompositions of $\mathcal{H}$ and F ($F = E' \wedge dt' + B'$) are performed, (E.4.15),[2] and they yield the usual identifications $(6)_1$ and $(6)_2$ in Ref. 26, but now with primes, $E'^i = F'^{i0}$, $B'^i = (1/2c)\varepsilon^{ijk0} F'_{jk}$. This discussion reveals that the (1+3) - splitting of spacetime, i.e., *the usual identifications* $(6)_1$, $(6)_2$ *in both frames*, are the real cause

that, although the authors[2] worked with exterior forms in the 4D spacetime, they obtained the same results for Minkowski's constitutive relations and for D and H ((E.4.25) and (E.4.26)[2]), i.e., for P and M, as those that were found by means of the 3-vectors and their UT, e.g., in Ref. 24.

However, as already discussed, e.g., in Refs. 27 and 11, and also in Ref. 12, *the (1+3) - splitting of spacetime and the usual identifications of components of, e.g., $F^{\alpha\beta}$ (implicitly taken in the standard basis) with components of the 3D* $\mathbf{E}$ *and* $\mathbf{B}$ *in both frames are synchronization dependent and they are meaningless in the 'r' synchronization.* We note that *different synchronizations are nothing else than different conventions and physics must not depend on conventions.* (In Ref. 26, Hehl (Eq. $(6)_2$) quotes Minkowski's identifications from Sec. 3[1] and states: 'We stress that the identifications in (6) are premetric, they are valid on any (well-behaved) differential manifold that can be split locally into time and space.' This means that Hehl considers that 1+3 decomposition, i.e., the space-time split, is also - premetric. But, if a nonstandard basis, the $\{r_\mu\}$ basis with the 'r' synchronization is used, i.e., when the appropriate metric is used, then it is not possible to make the usual identifications, that is, 1+3 decomposition, i.e., the space-time split. That metric is discussed in, e.g., the text below Eq. (14)[12]: '.. the components $g_{\mu\nu,r}$ of the metric tensor g_{ab} are $g_{ii,r}=0$, and all other components are $=1$.' Hence, both, the usual identifications (Eqs. (6)[26]), and 1+3 decomposition, i.e., the space-time split, are meaningful *only* when the Minkowski metric, e.g., $diag(1,-1,-1,-1)$, is used. Thus, these identifications depend on the chosen metric and therefore they are not premetric.

(For the $\{r_\mu\}$ basis see, for example, Refs. 27, 12.) In the $\{r_\mu\}$ basis, it holds that $F_r^{10}=E^1+cB^3-cB^2$. The relation for F_r^{10} shows that *the components have no definite physical meaning* since they are dependent on the chosen synchronization. Only in the $\{\gamma_\mu\}$ basis does it hold that $E^i=F^{i0}$, $P^i=\mathcal{M}^{i0}$, etc. According to that, the usual 3-vectors $\mathbf{E}$, $\mathbf{B}$, $\mathbf{D}$, $\mathbf{H}$, $\mathbf{P}$, $\mathbf{M}$, etc., where, e.g., $\mathbf{P}=\mathcal{M}^{10}\mathbf{i}+\mathcal{M}^{20}\mathbf{j}+\mathcal{M}^{30}\mathbf{k}$ ($\mathbf{i}$, $\mathbf{j}$, $\mathbf{k}$ are the unit 3-vectors) *have no definite physical meaning,* since the components $\mathcal{M}^{i0}$ are dependent on the chosen synchronization.)

Comparing, for example, the decompositions of F from Ref. 2 and 26 (the above quoted Eq. (34)[26]) with our decomposition $F=E\wedge v/c+(IcB)\cdot v/c$, Eq. (1), i.e., in some arbitrary basis $\{e_\mu\}$ (it can be the standard basis $\{\gamma_\mu\}$, or the $\{r_\mu\}$ basis with the 'r' synchronization, etc.) $F=(\delta^{\alpha\beta}{}_{\mu\nu}E_e^\mu v_e^\nu+c\varepsilon^{\alpha\beta\mu\nu}v_{e,\mu}B_{e,\nu})e_\alpha\wedge e_\beta$ ($\alpha,\beta,\mu,\nu=0,1,2,3$ and E_e^μ, B_e^μ, v_e^μ are the components in the $\{e_\mu\}$ basis), we see that there are important differences between them. All quantities, including v, in our relations are the 4D quantities and there is no (1+3) - splitting of spacetime, which means that they are more general than the decompositions from Ref. 2 and 26. If, for example, the magnetization 1-form M, Eq. (E.4.31),[2] would be written in the $\{\gamma_\mu\}$ basis then it would contain in the material frame *and in the laboratory frame* only spatial components and not the time component in contrast to the relation (21) (and (19)). Observe also that relations (13) and (14), or in the $\{\gamma_\mu\}$ basis (17) and (18), contain both the velocity of the observer v and the velocity of the considered medium

u, whereas there is only one velocity, the velocity of the considered medium, in the equations (E.4.25)-(E.4.31).[2]

Furthermore, the obvious similarity of the results with exterior forms[2] and the usual results with the 3-vectors and their UT, e.g. from Ref. 18 (or 24), can be nicely seen comparing Sec. E.4.5 under the title 'The experiments of Röntgen and Wilson & Wilson' in Ref. 2 and Secs. 5.2 and 5.3 in Ref. 18. It is visible from Eq. (E.4.49) and Fig. E.4.2[2] that in the matter-free region, region (2), there is only an electric field $E_{(2)}$, whereas there are *both* the electric field $E_{(1)}$ and *the induced magnetic field* $B_{(1)}$ in the slab moving with *constant* velocity. Completely the same result for that case is obtained in the third equation in (5.18) in Sec. 5.2,[18] where it is exclusively dealt with the 3-vectors and their UT. The same conclusion holds for the comparison of the treatments of the Wilsons' experiment, Sec. E.4.5[2] and Sec. 5.3.[18]

From the above discussion it is visible that either by using the 3-vectors and their UT, or by using the modern mathematical language, the exterior forms, always, in all previous treatments, it is obtained that, e.g. the electric field in one frame is 'seen' as slightly changed electric field *and the induced magnetic field* in the frame relatively moving with *constant* velocity. The same holds for the polarization and the magnetization and also for the electric and magnetic excitations.

6. The Physical Explanation of the Magnetoelectric Effect in Moving Media

The consideration in the previous sections shows that there are important differences in the form of Minkowski's constitutive relations (22), or equivalently the constitutive relations for $\mathbf{P}$ (23) and $\mathbf{M}$ (24), i.e., (25), and the constitutive relations for vectors P (13) and M (14). However, there is also a significant difference in physical interpretation of these constitutive relations. Particularly, this refers to the interpretation of the magnetoelectric effect for moving media.

In previous approaches it is considered that the term which describes how the magnetic field influences the polarization ($\gamma^2\varepsilon_0(\varepsilon_r - 1/\mu_r)\mathbf{U}\times\mathbf{B}$ in Eq. (25)) and the term which describes how the electric field influences the magnetization ($-\gamma^2\varepsilon_0(\varepsilon_r - 1/\mu_r)\mathbf{U}\times\mathbf{E}$ in Eq. (25)) are determined by the UT (6) for $\mathbf{E}$ and $\mathbf{B}$. Namely, according to the UT (6), in the rest frame of a moving medium the magnetic field 3-vector from the laboratory frame is 'seen' as a slightly changed magnetic field 3-vector and an *induced electric field 3-vector*. That induced electric field 3-vector interacts with the electric dipole moments 3-vectors by the interaction term $\mathbf{E}\cdot\mathbf{d}$ giving the polarization $\mathbf{P}$ as a 3-vector. Similarly, in the rest frame of a moving medium the electric field from the laboratory frame is 'seen' as a slightly changed electric field and an *induced magnetic field.* That induced magnetic field interacts with the magnetic dipole moments by the interaction term $\mathbf{B}\cdot\mathbf{m}$ giving the magnetization $\mathbf{M}$ (all are 3-vectors). In my opinion, it is very strange that, e.g., the magnetic field from a permanent magnet is 'seen' as an *induced electric field* in a relatively moving frame. What

is the physical reason which causes that one field becomes another field in a relatively moving frame? How can it be that a mere uniform moving transforms one field to another one? It cannot be that the transformations describe the physics, but, on the contrary, the physics has to describe how to get the correct transformations of the fields.

If the electric and magnetic fields are interpreted as vectors defined on the 4D spacetime then it is very natural to have that a magnetic field vector remains the magnetic field vector in a relatively moving frame. There is no physical cause that can change one field to another one in a relatively moving frame. According to the LT (8), an electric field vector transforms again to the electric field vector and similarly for a magnetic field vector. Hence, the explanation for the magnetoelectric effect in moving media is completely different in our approach.

Instead of dealing with the electric and magnetic dipole moments 3-vectors **d** and **m** we deal with the dipole moment bivector D, as a primary quantity for dipole moments, which does have a definite physical reality. The decomposition of D into the electric and magnetic dipole moments vectors d and m, respectively, and the unit time-like vector u/c, and the expressions for d and m, which are obtained from D and determined relative to u are

$$\begin{aligned} D &= d \wedge u/c + (mI) \cdot u/c^2, \\ d &= D \cdot u/c, \quad m = cI(D \wedge u/c); \end{aligned} \tag{27}$$

compare with (3) and (4). In this case the vector u is the bulk velocity vector of the medium as in (3) and (4). The interaction term in the Lagrangian for the interaction between the electromagnetic field F and the dipole moment bivector D can be written as a sum of two terms

$$\begin{aligned} L_{int} &= F \cdot D = (1/c^2)[-(E \cdot d + B \cdot m)(v \cdot u) + (E \cdot u)(v \cdot d) \\ &\quad +(B \cdot u)(v \cdot m)] + (1/c^3)[(E \wedge m - c^2 B \wedge d) \wedge v \wedge u]I. \end{aligned} \tag{28}$$

In the tensor formulation, the relations (27) and (28) are given by Eqs. (2) and (3), respectively, in Ref. 12 (they are first reported in Ref. 28). Observe that every term on the r.h.s. of (28) contains both velocities u and v. As seen from the last two terms they contain the direct interaction of E with m, and B with d. *These terms give the physical explanation for the existence of the magnetoelectric effect in moving media.* Moreover, there is no need for any transformation. We only need to represent E, d, B, m, u and v from (28) in the standard basis and then to choose the laboratory frame as our γ_0-frame. It can be seen from the discussion of Eq. (25)[12] that in the laboratory frame, as the γ_0-frame, and in the low velocity limit, we can neglect the contributions to L_{int} from the terms with d^0 and m^0; they are u^2/c^2 of the usual terms $E \cdot d$ or $B \cdot m$. Then, what remains from (28) is

$$L_{int} = -((E_i d^i) + (B_i m^i)) - (1/c^2)\varepsilon^{0ijk}(E_i m_k - c^2 B_i d_k)u_j. \tag{29}$$

This is, to order $0(u^2/c^2)$, relativistically correct expression with vectors for L_{int}. The last two terms that contain the direct interactions between E and m and

between B and d are not taken into account in any of the previous investigations of the magnetoelectric effect in moving media. They are u/c of the usual terms with the direct interaction of E with d and B with m. (The expression (29) is first reported in Ref. 29 in connection with the EDM searches.)

7. A Brief Discussion of the General Constitutive Relations

In this paper, the consideration is restricted to the constitutive relations and the magnetoelectric effect in moving media with homogeneous and isotropic electric and magnetic properties. For them, the rest frame constitutive relations are given by Eqs. (16). The general constitutive relations which cover dielectric, magnetic and magnetoelectric behavior are considered, e.g., in Ref. 26 and in more detail in Refs. 30, 2, 31 and 23. Mainly, e.g., Refs. 2, 26, 30, 31, these more general constitutive relations link the electromagnetic excitation $\mathcal{H}$ and F, $\mathcal{H}_{\alpha\beta} = \kappa_{\alpha\beta}{}^{\gamma\delta} F_{\gamma\delta}$ (our notation), Eq. (17),[30] where '$\kappa_{\alpha\beta}{}^{\gamma\delta}(x)$ is the twisted constitutive tensor of type $\begin{bmatrix} 2 \\ 2 \end{bmatrix}$' or, in another form, e.g., ${}^{*}\mathcal{H}^{\alpha\beta} = \chi^{\alpha\beta\gamma\delta} F_{\gamma\delta}$ (our notation), Eq. (21),[26] where ${}^{*}\mathcal{H}^{\alpha\beta} = (1/2)\varepsilon^{\alpha\beta\gamma\delta}\mathcal{H}_{\gamma\delta}$, $\chi^{\alpha\beta\gamma\delta}$ is 'a *constitutive tensor density* of rank 4 and weight +1, with the dimension $[\chi]$=1/resistance.' In Refs. 23, the general constitutive relations are established by expressing $\mathcal{M}$ as a linear function of the electromagnetic field F, $\mu_0 c \mathcal{M}_{\alpha\beta} = (1/2)\xi^{\gamma\delta}_{\alpha\beta} F_{\gamma\delta}$ (our notation), Eq. (2.45),[23] where '$\xi^{\gamma\delta}_{\alpha\beta}$ is the *general susceptibility tensor*, which is dimensionless because of the choice of the constant $\mu_0 c$.' However, in all these treatments, all quantities in the equations are not geometric quantities but components in the standard basis, which means that the components of F, $\mathcal{H}$, $\mathcal{M}$ are obtained by the usual identifications, i.e., they are considered to be the components of the 3-vectors **E** and **B**, **D** and **H**, **P** and **M**, respectively. This can be nicely seen from Eqs. (2.47) and (2.48)[23] and from their comparison with Eq. (2.9).[23] Hence, these relations are not so general relations and they are not premetric, as stated, e.g., in Ref. 26, because, as already mentioned in Sec. 5.3, the usual identifications and the space-time split are meaningless, e.g., in the $\{r_\mu\}$ basis with the 'r' synchronization.

In contrast with the usual covariant approach with coordinate-dependent quantities, all relations (1) - (5), (8), (11) - (16) and (27) - (28) are coordinate-free relations, which are written in terms of the abstract 4D geometric quantities.

8. Conclusions

The constitutive relations for P (13) and M (14) contain both velocity vectors u and v and thus they differ from all previous expressions. They are formulated in terms of coordinate-free quantities that correctly transform under the LT (8), whereas, as explained in Secs. 5 - 5.2, it is not the case with Minkowski's constitutive relations (23) and (24), or (25), with the 3-vectors that transform

according to the UT (6) and (7), which are not the LT. As discussed in Sec. 5.3, Minkowski's constitutive relations for P and M, (E.4.30) and (E.4.31), respectively, from Ref. 2, are obtained by the (1+3) - splitting of spacetime and consequently they are equivalent to the usual expressions with the 3-vectors (23) and (24). Hence, these relations,[2] which are obtained using exterior forms, also differ from our results (13) and (14). The differences that are quoted at the end of Sec. 5.1, points 1) - 3), could be used for the experimental examination and comparison of the results presented here and the constitutive relations which are obtained in the usual formulations either with the 3-vectors or with exterior forms. Furthermore, a completely new physical explanation of the magnetoelectric effect in moving media that is presented in Sec. 6, Eq. (28), or Eq. (29), offers the possibility for the experimental investigations of the magnetoelectric effect from the relativistically correct point of view. Regarding the importance of the magnetoelectric effect the results obtained in this paper could be enough important in different physical applications.

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